MONROE

Machine Methods

FOR THE EXTRACTION OF CUBE AND OTHER ROOTS

Monroe Calculating Machine Company, Inc.

ORANGE, NEW JERSEY

MACHINE METHOD
for the
Extraction of Cube and Other Roots

INTRODUCTION

EXTRACTION of cube and higher roots of numbers by the usual arithmetical methods has always necessitated numerous and more or less difficult and complicated calculations, involving a certain element of doubt as to the accuracy of the final result.

Many have the impression that there is no workable rule for finding the fifth and higher roots of numbers, even when such roots are exact.

Tables giving the cube root of numbers from 1 to 10,000 and tables of logarithms are available, but tables of this kind are awkward and mistakes can readily be made in reading the numbers or in interpolating between the numbers. The use of such tables is also limited because they can yield results correct only to a certain number of decimal places or digits.

A new and unusually simple machine method of extracting roots has recently been evolved that reduces these rather laborious and complicated calculations to a simple process of multiplication and division.

The method furnishes accuracy to the required number of decimal places or digits, and in addition affords a positive check on the work, even to the point that if an error is made, subsequent procedures will automatically correct the error.

The simplicity of the method is such that it can easily be mastered with a few minutes' practice, so that it will readily be recalled to mind when needed, even though used infrequently.

For Mathematicians, Actuaries, Engineers, Astronomers, Ballisticians, those engaged in Research and Statistical work, and in many other lines of endeavor, the extraction of roots is of daily occurrence. To those, and to students of advanced calculating practices, the following method for extracting roots should be both interesting and economically valuable.

OUTLINE OF METHOD

In starting the extraction of any root mentally approximate what the root might be. Particular care need not be taken as any approximation will do, although the closer the approximation the fewer will be the necessary calculations.

Example:

$$\sqrt[3]{128.53} = 5 \text{ (approximately)}$$

NOTE.—An approximation of 4, 4.5, 6 or even 12 would also serve.

The machine operations consist of using this mental approximation, A_1, in the formula given below to calculate a second approximation, A_2. A_2 is then substituted in the same formula to obtain a third approximation, A_3. The process is repeated until two approximations coincide. The last approximation will be the correct root of the number.

NOTE.—If the first or mental approximation is fairly accurate, it will seldom be necessary to perform more than three steps.

General Formula for Extracting All Roots

Let B = Base number of which the root is to be found.
A_1 = First approximation.
A_2 = Second approximation, etc.
i = Index designating the desired root.
$\doteq$ The equivalent of "the root is nearly approximated by."

then

$$\frac{\dfrac{B}{A_1^{\,i-1}} + A_1(i-1)}{i} = A_2, \text{ 2nd approximation,}$$

and

$$\frac{\dfrac{B}{A_2^{\,i-1}} + A_2(i-1)}{i} = A_3, \text{ 3rd approximation, etc.,}$$

repeated until two approximations coincide, thereby accurately determining the desired root.

PROBLEM

$$\sqrt[3]{128.53} = 5.04663$$

Arithmetical Substitution for Cube Root

If we take the first approximation, A_1, as 5, then,

$$\frac{\dfrac{128.53}{5^{\,3-1}} + 5(3-1)}{3} = A_2$$

or

$$\frac{\dfrac{128.53}{5^2} + (5 \times 2)}{3} \doteq 5.04706,\ A_2.$$

Substituting the value of A_2 in the same formula then,

$$\frac{\dfrac{128.53}{5.04706^2} + (5.04706 \times 2)}{3} \doteq 5.04663,\ A_3.$$

Substituting the new approximation, A_3 in the formula,

$$\frac{\dfrac{128.53}{5.04663^2} + (5.04663 \times 2)}{3} = 5.04663,$$

which is positive proof that the approximation A_3, 5.04663, is the cube root of 128.53, correct to five decimal places.

Monroe Method for Cube Root

Decimal set-up for roots of five decimal places.

Upper dials	5
Keyboard	5
Lower dials	10

Use same decimal ratio for roots to other required decimal places.

All adding / subtracting in "6" position

SECOND APPROXIMATION

Step 1. Set 5.00000 on the keyboard and multiply by 5.00000. Set the base number, 128.53000, on the keyboard; mentally note all figures in the lower dials directly above the base number (in this case 25.) and depress the plus bar once.

2. Set on the keyboard the number just noted, 25, and depress the minus bar once. (The above operation leaves the machine in position to divide without writing intermediate steps.) Clear the upper dials and perform the division.

3. Copy the result, 5.14120 to the keyboard and add. Set 5.00000 on the keyboard and multiply by 2.00000. Clear the upper dials. Set 3.00000 on the keyboard and divide. Result is 5.04706, the second approximation, A_2. Clear the lower dials and make a notation of this approximation.

THIRD APPROXIMATION

Step 1. Set 5.04706 on the keyboard and with plus bar return the upper dials to zeros. Set the base number, 128.53000, on the keyboard; mentally note all figures in the lower dials directly above the base number (in this case 25.47) and depress the plus bar once.

2. Set on the keyboard the number just noted, 25.47, also copy to the three remaining columns on the right of the keyboard the figures in the lower dials directly above these columns (keyboard now reads 25.47281) and depress the minus bar once; then divide. Clear the lower dials.

3. Copy the result, 5.04577, to the keyboard and add. Set 5.04706 on the keyboard and multiply by 2.00000. Clear the upper dials. Set 3.00000 on the keyboard and divide. Result is 5.04663, the third approximation, A_3. Make a note of this approximation.

FOURTH APPROXIMATION

Step 1. Set 5.04663 on the keyboard and with plus bar return the upper dials to zeros. Set the base number, 128.53000, on the keyboard; mentally note all figures in the lower dials directly above the base number (in this case 25.46) and depress the plus bar once.

2. Set on the keyboard the number just noted, 25.46, also copy to the three remaining columns on the right of the keyboard the figures in the lower dials directly above these columns, (keyboard now reads 25.46847) and depress the minus bar once; then divide. The result, *5.04663,* coincides with the third approximation, A_3 which automatically proves that this amount is the cube root of 128.53.

NOTE.—It is evident that Step 3 is unnecessary as this or any subsequent steps could only result in the same root, *5.04663.*

PROBLEM

$$\sqrt[5]{885.50} = 3.885417$$

Arithmetical Substitution for Fifth Root

Mentally approximate what the root might be. As $3^5 = 243$ and $4^5 = 1024$ it is evident that the root must be a little under 4, so for the purpose of this illustration we may use 3.9 as A_1 although any other approximation would do.

Using the first approximation, A_1, as 3.9, then

$$\frac{\frac{885.50}{3.9^{5-1}} + 3.9\,(5-1)}{5} = A_2$$

or

$$\frac{\frac{885.50}{3.9^4} + (3.9 \times 4)}{5} \doteq 3.885526,\ A_2.$$

Substituting the value of A_2 in the same formula, then

$$\frac{\frac{885.50}{3.885526^4} + (3.885526 \times 4)}{5} \doteq 3.885417,\ A_3.$$

Substituting the new approximation A_3 in the formula,

$$\frac{\frac{885.50}{3.885417^4} + (3.885417 \times 4)}{5} = 3.885417,$$

which proves that the approximation, A_3, 3.885417, is the fifth root of 885.50 correct to six decimal places.

Monroe Method for Fifth Root

Decimal Set-up for Roots of Six Decimal Places

Upper dials	6
Keyboard	6
Lower dials	12

All adding / subtracting in "7" position

SECOND APPROXIMATION

Step 1. Set 3.900000 on the keyboard and multiply by 3.900000. Copy 15.210000 to the keyboard and subtract to prove. Clear the upper dials. Multiply by 15.210000. Set the base number, 885.500000, on the keyboard; mentally note all figures in the lower dials directly above the base number (in this case 231.3) and depress the plus bar once.

2. Set on the keyboard the number just noted, 231.3, and also copy to the remaining columns on the right of the keyboard the figures in the lower dials directly above these columns (keyboard now reads 231.344100) and depress the minus bar once. (This operation leaves the machine in position to divide without writing intermediate steps.) Clear the upper dials and divide. Clear the lower dials.

3. Copy the result, 3.827631, to the keyboard and add. Clear the upper dials. Set 3.900000 on the keyboard and multiply by 4.000000. Clear the upper dials. Set 5.000000 on the keyboard and divide. Result is 3.885526, the second approximation, A_2. Make a notation of this approximation and clear the lower dials.

THIRD APPROXIMATION

Step 1. Set 3.885526 on the keyboard and with the plus bar return the upper dials to zero. Copy 15.097312 to the keyboard and subtract to prove. Clear the upper dials and multiply by 15.097312. Set the base number, 885.500000, on the keyboard; mentally note all figures in the lower dials directly above the base number (in this case, 227.9) and depress the plus bar once.

Step 2. Set on the keyboard the number just noted, 227.9, and also copy to the remaining columns on the right of the keyboard the figures in the lower dials directly above these columns (keyboard now reads 227.928829) and depress the minus bar once. Clear the upper dials and divide. Clear the lower dials.

3. Copy the result, 3.884984, to the keyboard and add. Clear the upper dials. Set 3.885526 on the keyboard and multiply by 4.000000. Clear the upper dials and divide by 5.000000. Result is 3.885417, the third approximation, A_3. Make a notation of this approximation and clear the lower dials.

FOURTH APPROXIMATION

Step 1. Set 3.885417 on the keyboard and with the plus bar return the upper dials to zero. Copy 15.096465 to the keyboard and subtract to prove. Clear the upper dials and multiply by 15.096465. Set the base number, 885.500000, on the keyboard; mentally note all figures in the lower dials directly above the base number (in this case, 227.9) and depress the plus bar once.

2. Set on the keyboard the number just noted, 227.9, and also copy to the remaining columns on the right of the keyboard the figures in the lower dials directly above these columns (keyboard now reads 227.903255) and depress the minus bar once. Clear the upper dials and divide. Clear the lower dials.

3. Copy the result, 3.885420, to the keyboard and add. Clear the upper dials. Set 3.885417 on the keyboard and multiply by 4.000000. Clear the upper dials and divide by 5.000000. The result, *3.885147*, coincides with the third approximation, A_3, thereby automatically proving that the third approximation was the fifth root of 885.50, correct to six decimal places.

For further instructions or help with any figuring problems, users of the Monroe Adding-Calculator are invited to get in touch with the nearest local office. Branches are maintained in all the principal cities in the country. Monroe representatives are qualified to recommend the most efficient methods for the handling of figure work and will gladly be of service.

MONROE

COMPLETE FIGURING SERVICE

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